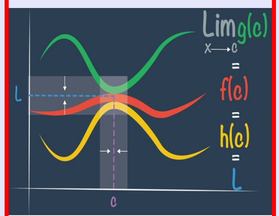
Calculus I

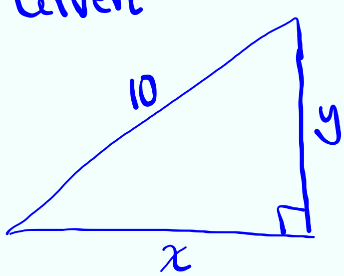
Lecture 21



Feb 19-8:47 AM

Class QZ 18

Given



x is decreasing at 4cm/min.

How fast is y changing when $y = 8$cm.

$\boxed{\text{Increasing}}$ or Decreasing?

$x^2 + y^2 = 10^2$

$x^2 + 8^2 = 10^2$

$x = 6$

$$\to 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$(6)(-4) + 8\frac{dy}{dt} = 0 \to \boxed{\frac{dy}{dt}\ 3\text{cm/min}}$$

Apr 30-11:01 AM

Rolle's Theorem

If

1) $f(x)$ is continuous on $[a,b]$,

2) $f(x)$ is differentiable on (a,b), and

3) $f(a) = f(b)$

Then there is a number c in (a,b) such that $f'(c) = 0$

May 5-8:53 AM

Given $f(x) = 3x^2 - 12x + 5$ on $[1,3]$

$f(x)$ is a polynomial function

1) $f(x)$ is cont. on $[1,3]$

2) $f(x)$ is diff. on $(1,3)$

3) $f(1) = 3(1)^2 - 12(1) + 5 = 3 - 12 + 5 = -4$

$f(3) = 3(3)^2 - 12(3) + 5 = 27 - 36 + 5 = -4$

by Rolle's thrm, there is a number c in $(1,3)$ such that $f'(c) = 0$.

$f(x) = 3x^2 - 12x + 5$

$f'(x) = 6x - 12$

$f'(c) = 6c - 12$

$f'(c) = 0$

$6c - 12 = 0$

$\boxed{c = 2}$

May 5-8:55 AM

$f(x) = \sin x \quad , [0, \pi]$

1) $f(x)$ is Cont. $(-\infty, \infty) \rightarrow f(x)$ is Cont. on $[0, \pi]$

2) $f'(x) = \cos x \quad , \quad f'(x)$ is Cont. on $(0, \pi)$

$f(x)$ is diff. on $(0, \pi)$

3) $f(0) = \sin 0 = 0 \quad , \quad f(\pi) = \sin \pi = 0$

$f(0) = f(\pi)$

by Rolle's Thrm, there is a number C in $(0, \pi)$ Such that $f'(c) = 0$

$f(x) = \sin x$

$f'(x) = \cos x \qquad f'(c) = \cos C$

$\cos C = 0 \rightarrow \boxed{C = \frac{\pi}{2}}$

May 5-9:01 AM

$f(x) = \sqrt{x} - \frac{1}{3}x \quad , [0, 9]$

$\sqrt{x}$ is defined $[0, \infty)$

x is " $(-\infty, \infty)$

1) $f(x)$ is Cont. in $[0, 9]$

2) $f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3} \qquad f'(x)$ is Cont. in $(0, 9)$

$f(x)$ is diff. in $(0, 9)$

3) $f(0) = 0 \quad , \quad f(9) = \sqrt{9} - \frac{1}{3}(9) = 0$

$f(0) = f(9)$

By Rolle's thrm, there is a number c in $(0, 9)$ Such that $f'(c) = 0$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3} \qquad f'(c) = \frac{1}{2\sqrt{c}} - \frac{1}{3}$

$f'(c) = 0 \qquad \frac{1}{2\sqrt{c}} - \frac{1}{3} = 0 \qquad \frac{1}{2\sqrt{c}} = \frac{1}{3}$

Cross Multiply $\qquad 2\sqrt{c} = 3$

$\sqrt{c} = \frac{3}{2}$

$c = \frac{9}{4} = \boxed{2.25}$

in $(0, 9)$

May 5-9:06 AM

Mean - Value Theorem

1) $f(x)$ is cont. on $[a,b]$,

2) $f(x)$ is diff. on (a,b)

then there is a number C in (a,b)

such that $f'(c) = \dfrac{f(b) - f(a)}{b-a}$.

May 5-9:13 AM

$f(x) = 2x^2 - 3x + 1$, $[0,2]$

1) $f(x)$ is polynomial $\to$ cont. on $[0,2]$

2) $f(x)$ is polynomial $\to$ diff. on $(0,2)$

By MVT, there is a number C in $(0,2)$

such that $f'(c) = \dfrac{f(2) - f(0)}{2-0}$

$f(x) = 2x^2 - 3x + 1$

$f'(x) = 4x - 3$

$4c - 3 = \dfrac{3-1}{2-0}$

$f(2) = 2(2)^2 - 3(2) + 1 = 3$

$f(0) = 2(0)^2 - 3(0) + 1 = 1$

$4c - 3 = 1 \to \boxed{c = 1}$

May 5-9:16 AM

$f(x) = \frac{1}{x}$, $[1,3]$

Domain $(-\infty,0) \cup (0,\infty)$

$f'(x) = \frac{-1}{x^2}$

1) $f(x)$ is Cont. on $[1,3]$

2) $f(x)$ is diff. on $(1,3)$

By MVT, there is a number C in $(1,3)$

Such that $f'(c) = \frac{f(3) - f(1)}{3-1}$

$$\frac{-1}{c^2} = \frac{\frac{1}{3} - 1}{3-1}$$

$$\frac{-1}{c^2} = \frac{-\frac{2}{3}}{2} \qquad \frac{-1}{c^2} = \frac{-1}{3}$$

$$c^2 = 3$$

$$c = \pm\sqrt{3}$$

$$\boxed{c = \sqrt{3}}$$

May 5-9:20 AM

$f(x) = x^3 - 3x + 2$, $[-2,2]$ $\quad f'(x) = 3x^2 - 3$

$f(x)$ is a polynomial function $\quad f(2) = 2^3 - 3(2) + 2 = 4$

1) $f(x)$ is Cont. everywhere $\quad f(-2) = (-2)^3 - 3(-2) + 2 = 0$

2) $f(x)$ is diff.

by MVT, there is a number C in $(-2,2)$

Such that $f'(c) = \frac{f(2) - f(-2)}{2-(-2)}$

$$3c^2 - 3 = \frac{4 - 0}{4}$$

$$3c^2 - 3 = 1 \qquad 3c^2 = 4 \qquad c^2 = \frac{4}{3}$$

$$c = \pm\frac{2}{\sqrt{3}}$$

$$c \approx \boxed{\pm 1.2}$$

May 5-9:25 AM

$f(x) = x^5 - 5x$

Domain $(-\infty, \infty)$

Y-Int $(0,0)$

x-Int $(0,0)$

$(\sqrt[4]{5}, 0)$, $(-\sqrt[4]{5}, 0)$

$f(x) = 0$

$x^5 - 5x = 0$

$x(x^4 - 5) = 0$

$x = 0$ $\quad$ $x^4 = 5$

$x = \pm\sqrt[4]{5}$

$f(-x) = (-x)^5 - 5(-x) = -x^5 + 5x = -(x^5 - 5x) = -f(x)$

$f(x)$ is an odd function

Symmetric w/t the origin

$f'(x) = 5x^4 - 5$

$f'(x) = 0$

$x = \pm 1$

$f(1) = -4$ $\quad$ $f(-1) = 4$

$f''(x) = 20x^3$

$f''(x) = 0$

$x = 0$

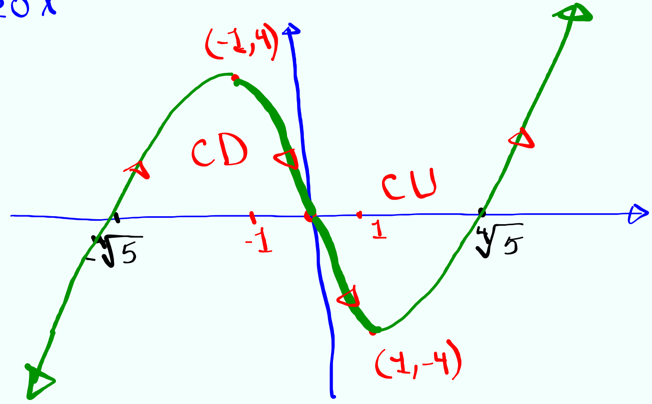
$f(0) = 0$

May 5-9:34 AM

x	$-\infty$	-1		0		1	∞
$f'(x)$	+	•	−		−	•	+
$f''(x)$	−		−	•	+		+
$f(x)$	↗		↘		↘		↗

$f'(x) = 5x^4 - 5 = 5(x^4 - 1) = 5(x^2+1)(x^2-1) = 5(x^2+1)(x+1)(x-1)$

$f''(x) = 20x^3$



May 5-9:40 AM

$f(x) = \dfrac{x^2}{x^2+3}$

1) Domain $(-\infty, \infty)$

2) Y-Int $(0,0)$

3) x-Int $(0,0)$

4) $\lim\limits_{x \to \infty} f(x) = 1$

5) $\lim\limits_{x \to -\infty} f(x) = 1$

6) Show $f(x)$ is an even function

$f(-x) = \dfrac{(-x)^2}{(-x)^2+3} = \dfrac{x^2}{x^2+3} = f(x)$ even function

Symmetric w/t Y-axis

7) $f'(x) = \dfrac{6x}{(x^2+3)^2}$

$f'(x) = 0 \quad x = 0$

$(0,0)$

8) $f''(x) = \dfrac{-18(x^2-1)}{(x^2+3)^3}$

$f''(x) = 0 \rightarrow x = \pm 1$

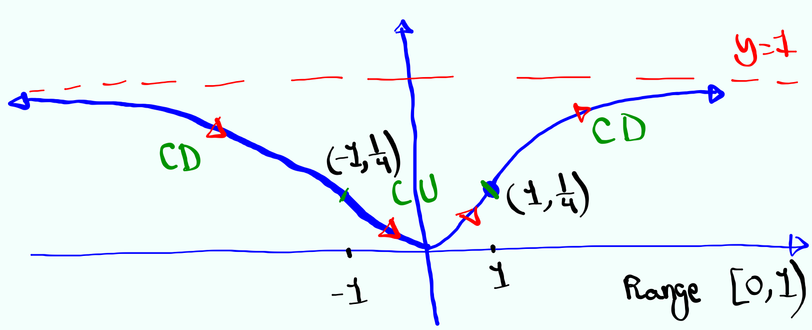
$(1, \frac{1}{4})$, $(-1, \frac{1}{4})$

May 5-9:47 AM

$f'(x) = \dfrac{6x}{(x^2+3)^2}$

$f''(x) = \dfrac{-18(x+1)(x-1)}{(x^2+3)^3}$

x	$-\infty$	-1		0		1	∞
$f'(x)$	$-$		$-$	0	$+$		$+$
$f''(x)$	$-$	0	$+$		$+$	0	$-$
$f(x)$							



Range $[0, 1)$

May 5-9:59 AM

Two positive numbers have a Sum of 16.

x & y $\qquad$ $x+y=16$

$x>0,\ y>0$ $\qquad$ $y=16-x$

what is the Smallest value of the Sum of their Squares? $\rightarrow$ Minimum

$x^2 + y^2 = x^2+(16-x)^2$

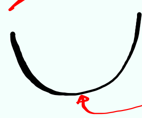
$f(x)= x^2+(16-x)^2$

$f'(x) = 2x + 2(16-x)^1\cdot(-1) = 2x - 2(16-x)$

$= 2x - 32 + 2x$

$= 4x - 32$

$f''(x) = 4 > 0$



min. $\rightarrow f'(x)=0$

$4x-32=0$

$x=8$

$y=16-x$

$y=8$

$x^2+y^2=$

$8^2+8^2=\boxed{128}$

Smallest value of sum of their sqrs.

May 5-10:13 AM

Introduction to integration

Reverse of differentiation

Assuming $f'(x)$ is Continuous

$$\int f'(x)\,dx = f(x) + C$$

Integral $\qquad$ integrand

$$\frac{d}{dx}\left[f(x)+C\right] = f'(x)$$

1) $\int 2\,dx = 2x + C$ $\qquad$ $\frac{d}{dx}\left[2x+C\right]=2$

2) $\int 2x\,dx = x^2 + C$ $\qquad$ $\frac{d}{dx}\left[x^2+C\right]=2x$

May 5-10:22 AM

Power Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1.$$

$$\int x^3 dx = \frac{x^{3+1}}{3+1} + C = \boxed{\frac{1}{4}x^4 + C}$$

$$\frac{d}{dx}\left[\frac{1}{4}x^4 + C\right] = \frac{1}{4}\cdot 4x^3 + 0 = \boxed{x^3}$$

$$\int \sqrt{x}\, dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \boxed{\frac{2}{3}x^{\frac{3}{2}} + C}$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

May 5-10:27 AM

Integration Properties:

1) $\int c\, dx = cx + K$

2) $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1$

3) $\int c\, f(x)\, dx = c\int f(x)\, dx$

4) $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

May 5-10:35 AM

$$\int 6x^2\,dx = 6\int x^2\,dx$$

$$= 6\cdot\frac{x^{2+1}}{2+1} + C$$

$$= \frac{6x^3}{3} + C = \boxed{2x^3 + C}$$

$$\int \frac{1}{5}\sqrt[4]{x^3}\,dx = \frac{1}{5}\int x^{\frac{3}{4}}\,dx = \frac{1}{5}\cdot\frac{x^{\frac{3}{4}+1}}{\frac{3}{4}+1} + C$$

$$= \frac{1}{5}\cdot\frac{x^{\frac{7}{4}}}{\frac{7}{4}} + C$$

$$= \frac{1}{5}\cdot\frac{4x^{\frac{7}{4}}}{7} + C$$

$$= \boxed{\frac{4}{35}x^{7/4} + C}$$

May 5-10:38 AM

$$\int (12x^3 + 2\cos x)\,dx$$

Indefinite integral

$$= \int 12x^3\,dx + \int 2\cos x\,dx$$

$$= 12\cdot\frac{x^4}{4} + 2\cdot\sin x + C$$

$$= \boxed{3x^4 + 2\sin x + C}$$

$$\int (\cos x - \sec^2 x)\,dx$$

$$= \int \cos x\,dx - \int \sec^2 x\,dx = \boxed{\sin x - \tan x + C}$$

May 5-10:45 AM

Definite integral

Assuming $f'(x)$ is Continuous on $[a,b]$

$$\int_a^b f'(x)\,dx = f(x)\Big|_a^b = f(b) - f(a)$$

$$\int_1^2 2x\,dx = x^2\Big|_1^2 = (2)^2 - (1)^2 = 3$$

$$\int_1^4 4\sqrt{x}\,dx = 4\int_1^4 x^{\frac{1}{2}}\,dx = 4\cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_1^4$$

$$= 4\cdot\frac{2}{3}x^{\frac{3}{2}}\Big|_1^4$$

$$= \frac{8}{3}x\sqrt{x}\Big|_1^4 = \frac{8}{3}\left[4\sqrt{4} - 1\sqrt{1}\right]$$

$$= \frac{8}{3}[8-1]$$

$$= \frac{8}{3}\cdot 7 = \boxed{\frac{56}{3}}$$

May 5-10:50 AM

Evaluate $\displaystyle\int_0^{\pi/2} \cos x\,dx = \sin x\Big|_0^{\pi/2}$

$$= \sin\frac{\pi}{2} - \sin 0$$

$$= 1 - 0 = \boxed{1}$$

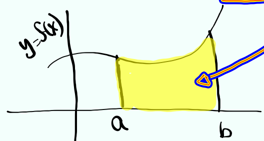
Evaluate $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\,dx = \tan x\Big|_{-\pi/4}^{\pi/4}$

$$= \tan\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)$$

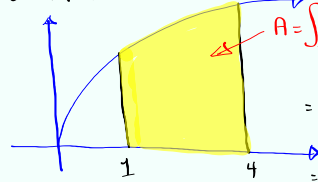
$$= 1 - (-1) = \boxed{2}$$

May 5-10:57 AM

If $f(x)$ is Cont. on $[a,b]$ and $f(x)\geq 0$ on $[a,b]$, then the area under $f(x)$, above x-axis from $x=a$ to $x=b$ is given by $\boxed{\int_a^b f(x)\,dx.}$



Find the area bounded by $x=1$, $x=4$, x-axis, and $f(x)=\sqrt{x}$.



$$A=\int_1^4 \sqrt{x}\,dx=\int_1^4 x^{\frac{1}{2}}dx$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^4$$

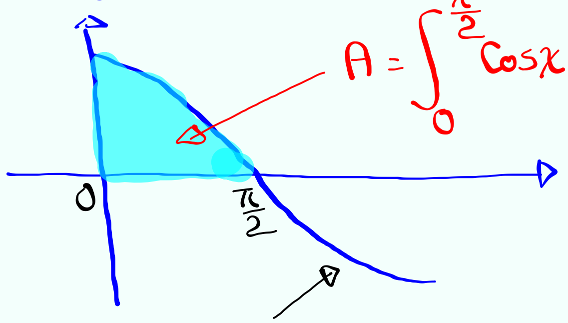
$$=\frac{2}{3}\cdot x\sqrt{x}\Big|_1^4$$

$$=\frac{2}{3}\left[4\sqrt{4}-1\sqrt{1}\right]$$

$$=\boxed{\frac{14}{3}}$$

May 5-11:03 AM

Find the shaded area below



y = Cos x

$$A=\int_0^{\frac{\pi}{2}} \cos x\,dx=\sin x\Big|_0^{\frac{\pi}{2}}$$

$$=\sin\frac{\pi}{2}-\sin 0$$

$$=1-0=\boxed{1}$$

Google Critical numbers &

Inflection points

May 5-11:10 AM